

**Lepton flavour violating pion decay $\pi^+ \rightarrow \mu^- \nu_\mu e^+ e^+$
and the $SU(3)_C \otimes SU(3)_L \otimes U(1)$ model****Hoang Ngoc Long¹***Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany***Abstract**

In the framework of the minimal $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ model, the lepton-flavour-violating decay $\pi^+ \rightarrow \mu^- \nu_\mu e^+ e^+$ is calculated without directly invoking lepton mixing. The branching ratio for this rare pion decay mode is found to be much smaller than the current experimental upper limit. Dropping out anomalous interactions, this result coincides with the previous calculation .

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At present, neutrinos are presumably massive and mixed as indicated in various experiments: SuperKamiokande [1] and others [2]. This significant deviation from the standard model (SM) calls for its extension. The models based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ (3 3 1) gauge group [3, 4] are one of the most popular in such extensions beyond SM. The SM assumes lepton-flavour-number conservation, and its observed violation would be a clear indication of new physics. In the 3 3 1 models the lepton-flavour number is not conserved, and these models have motivated a variety of dedicated sensitive searches for rare decay modes of muons and kaons and for neutrino oscillations [5]. It is known that the muon system is one of the best places to search for lepton flavour violation, compared with the others. The “wrong” muon decay $\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu$ is widely used to put a lower bound on the singly-charged bilepton mass ($M_Y \geq 230$ GeV) [6].

In this work we pay attention to the lepton-flavour-violating pion decay $\pi^+ \rightarrow \mu^- \nu_\mu e^+ e^+$. The upper limit in its branching ratio is given $R \leq 1.6 \times 10^{-6}$ at 90 % confidence level [7, 8]. By suggesting the lepton mixing or horizontal interactions, the above decay has been studied theoretically in Ref. [9]. However, this decay may be described by the minimal 3 3 1 model in simple manner without directly invoking lepton mixing.

To start, we firstly give some basic elements of the model (for more details see [10]). Three lepton components of each family are in one triplet,

$$f_L^a = (\nu_L^a, l_L^a, (l^c)_L^a)^T,$$

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where $a = 1, 2, 3$ is the family index. Under $SU(3)_L$, two of the three quark families transform as antitriplet and one family transforms as triplet,

$$Q_{iL} = \begin{pmatrix} d_{iL} \\ -u_{iL} \\ D_{iL} \end{pmatrix}, (i = 1, 2), \quad Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix}.$$

The right-handed quarks are singlets under $SU(3)_L$. The exotic quarks T and D_i have an electric charge $+5/3$ and $-4/3$, respectively.

There are five new gauge bosons: the Z' and the charged bileptons with lepton number $L = \pm 2$, which are identified as follows: $\sqrt{2} Y_\mu^- = W_\mu^4 - iW_\mu^5$, $\sqrt{2} X_\mu^{--} = W_\mu^6 - iW_\mu^7$, and their couplings to leptons are given by [11]:

$$\mathcal{L}_l^{CC} = -\frac{g}{2\sqrt{2}} \left[\bar{\nu} \gamma^\mu (1 - \gamma_5) C \bar{l}^T Y_\mu^- - \bar{l} \gamma^\mu \gamma_5 C \bar{l}^T X_\mu^{--} + \text{h.c.} \right]. \quad (1)$$

The interactions among the charged vector fields with quarks are

$$\begin{aligned} \mathcal{L}_q^{CC} = & -\frac{g}{\sqrt{2}} [(\bar{u}_{3L} \gamma^\mu d_{3L} + \bar{u}_{iL} \gamma^\mu d_{iL}) W_\mu^+ + (\bar{T}_L \gamma^\mu d_{3L} + \bar{u}_{iL} \gamma^\mu D_{iL}) X_\mu^{++} \\ & + (\bar{u}_{3L} \gamma^\mu T_L - \bar{D}_{iL} \gamma^\mu d_{iL}) Y_\mu^- + \text{h.c.}]. \end{aligned} \quad (2)$$

It is to be noted that the vector currents coupled to X^{--} , X^{++} vanish due to Fermi statistics, and the exotic quarks interact with ordinary ones only via the bileptons and non-SM Higgs bosons.

The current experimental lower bound on the exotic quark mass is 200 GeV [12], while the lower bound on the bilepton mass is in the range of 300 GeV.

To deal with the above process we also need the coupling constants of the bileptons X , Y to the SM weak-vector boson W . In the notation of Refs. [13] it is: $CWXY = \frac{g}{\sqrt{2}}$.

Now we start with the decay

$$\pi^+(K) \rightarrow \mu^-(p) \nu_\mu(q) e^+(k_1) e^+(k_2), \quad (3)$$

where the letters in parentheses stand for the momenta of the particles. We assume that the Higgs bosons responsible for lepton-flavour-violating interactions as well as the exotic quarks are much heavier than the standard model W boson. Hence the contributions from the exotic quarks and non SM Higgs bosons are suppressed. With new gauge bosons carrying lepton-number $L = 2$, the process (3) can be described simply by the Feynman diagram depicted in Fig. 1.

For small momentum-transfer ($q^2 \ll m_W^2, M_X^2, M_Y^2$), as is the case here, the matrix element for this process is found to be

$$\begin{aligned} \mathcal{M}_{fi} = & 2 \frac{G_F^2 f_\pi m_W^2}{M_X^2 M_Y^2} [-(P + K)_\beta K_\gamma + (K + L)_\gamma K_\beta + (-L + P)_\gamma K_\beta g_{\beta\gamma}] \\ & \times \bar{u}_{\nu_\mu}(q) \gamma^\beta (1 - \gamma_5) C \bar{u}_\mu^T(p) \cdot v_e^T(k_1) C \gamma^\gamma \gamma_5 v_e(k_2), \end{aligned} \quad (4)$$

where the following combinations of four vectors are introduced

$$P = k_1 + k_2, \quad Q = k_1 - k_2, \quad L = p + q, \quad N = p - q, \quad K = P + L. \quad (5)$$

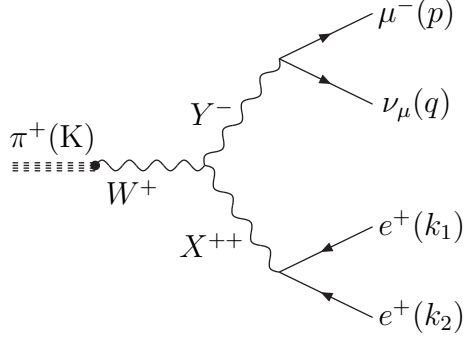


Figure 1: Feynman diagram for the decay $\pi^+(K) \rightarrow \mu^-(p) \nu_\mu(q) e^+(k_1) e^+(k_2)$ in the 3 3 1 model

The squared matrix element is given by

$$\begin{aligned}
|\mathcal{M}_{fi}|^2 &= 128 \frac{G_F^4 f_\pi^2 m_W^4}{M_X^4 M_Y^4} C_{\beta\gamma} C_{\beta'\gamma'} \\
&\times \left[p^\beta q^{\beta'} + p^{\beta'} q^\beta - g^{\beta\beta'} (p \cdot q) + i \varepsilon^{\beta\beta' mn} p_m q_n \right] \\
&\times \left[k_1^\gamma k_2^{\gamma'} + k_1^{\gamma'} k_2^\gamma - g^{\gamma\gamma'} (k_1 \cdot k_2 - m_e^2) \right], \tag{6}
\end{aligned}$$

where the notation $C_{\beta\gamma} \equiv [-(P+K)_\beta K_\gamma + (K+L)_\gamma K_\beta + (-L+P) \cdot K g_{\beta\gamma}]$ is used.

In order to describe the kinematics of the decay, we introduce the following vectors: Let \vec{v} be a unit vector along the direction of flight of the dipositron in the π^+ rest system (Σ_π), \vec{a} be a unit vector along the projection of the three-momentum of the e^+ in the $e^+ e^+$ center-of-mass system (Σ_{2e}) perpendicular to \vec{v} , and \vec{b} be a unit vector along the projection of the three-momentum of the μ^- in the $\mu^- \nu_\mu$ center-of-mass system ($\Sigma_{\mu\nu}$) perpendicular to $-\vec{v}$. Then the kinematics of this decay is similar to the one given in [14], which consists of five variables: $s_e \equiv P^2$, $s_\mu \equiv L^2$, and three angles: (i) θ_e , the angle of the e^+ in Σ_{2e} with respect to the dipositron line of flight in Σ_π , (ii) θ_μ , the angle of the μ^- in $\Sigma_{\mu\nu}$ with respect to the $\mu\nu_\mu$ line of flight in Σ_π , and (iii) ϕ , the angle between the plane formed by the positrons in Σ_π and the corresponding plane formed by the μ^-, ν_μ . The angles θ_e, θ_μ and ϕ are shown in Fig. 2.

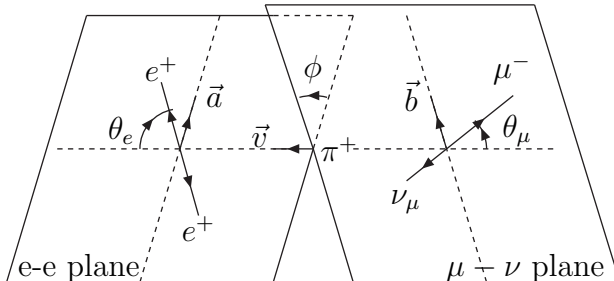


Figure 2: Illustration of the angles θ_e , θ_μ and ϕ

Then the decay width for the pion decay (3) is written as

$$d\Gamma = \left(\frac{1}{2}\right) \frac{1}{2^{14}\pi^6 m_\pi^3} \sum_{spins} |\mathcal{M}_{fi}|^2 (1 - z_\mu) \sigma_e X ds_e ds_\mu d(\cos_e) d(\cos_\mu) d\phi. \quad (7)$$

In (7), $\left(\frac{1}{2}\right)$ is the statistical factor indicating that two (identical) positrons in the final state [15]. With the above definitions we have the following scalar products

$$\begin{aligned} Q^2 &= 4m_e^2 - s_e, \quad N^2 = 2m_\mu^2 - s_\mu, \quad K^2 = m_\pi^2, \quad L.N = m_\mu^2, \\ P.L &= \frac{1}{2}(m_e^2 - s_e - s_\mu), \quad P.N = z_\mu P.L + (1 - z_\mu)X \cos \theta_\mu, \quad Q.L = \sigma_e X \cos \theta_e, \\ Q.N &= z_\mu Q.L + \sigma_e(1 - z_\mu)P.L \cos \theta_e \cos \theta_\mu - (s_e s_\mu)^{1/2} \sin \theta_e \sin \theta_\mu \cos \phi, \\ d &\equiv \varepsilon^{\mu\nu\alpha\beta} L_\mu N_\nu P_\alpha Q_\beta = -(s_e s_\mu)^{1/2} \sigma_e (1 - z_\mu) X \sin \theta_e \sin \theta_\mu \sin \phi, \end{aligned} \quad (8)$$

where

$$z_\mu \equiv \frac{m_\mu^2}{s_\mu}, \quad \sigma_e \equiv \left(1 - \frac{4m_e^2}{s_e}\right)^{1/2}, \quad X \equiv ((P.L)^2 - s_e s_\mu)^{1/2},$$

and m_e, m_μ, m_π stand for masses of the electron, the muon and the pion, respectively. The range of the variables is

$$\begin{aligned} 4m_e^2 &\leq s_e \leq (m_\pi - m_\mu)^2, \\ m_\mu^2 &\leq s_\mu \leq (m_\pi - \sqrt{s_e})^2, \\ 0 &\leq \theta_e, \theta_\mu \leq \pi, \quad 0 \leq \phi \leq 2\pi. \end{aligned} \quad (9)$$

It is to be noted that an imaginary part of $|\mathcal{M}_{fi}|^2$ connected with pseudotensor d is *linear* in $\sin \phi$, i.e. no such a term like $Q.Nd$, hence it will be removed after integration over the angle ϕ . In resulting we get the decay width being a real number, as it has to be.

The integrations over the angles can be carried out analytically by using Mathematica. The numerical integrations over the effective masses squared s_e and s_μ are carried out by employing the Monte Carlo routine VEGAS [16]. After changing to dimensionless parameters $x_e = \frac{s_e}{m_\pi^2}$, $y_\mu = \frac{s_\mu}{m_\pi^2}$, we get the decay width

$$\Gamma(\pi^+ \rightarrow \mu^- \nu_\mu e^+ e^+) = \frac{G_F^4 f_\pi^2 m_W^4 m_\pi^{11} N}{256\pi^6 M_X^4 M_Y^4}, \quad (10)$$

where N is numerically evaluated, $N = 6.17 \times 10^{-6}$. We recall that the main (99.987 %) decay mode of the π^+ is well-known

$$\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu) = \frac{G_F^2 f_\pi^2 m_\mu^2}{8\pi m_\pi^3} (m_\pi^2 - m_\mu^2)^2 \simeq 2.63 \times 10^{-17} \text{ GeV}. \quad (11)$$

From (10) and (11) we get the branching ratio

$$\begin{aligned}
R_\pi &= \frac{\Gamma(\pi^+ \rightarrow \mu^- \nu_\mu e^+ e^+)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} = \frac{6.17 \times 10^{-6} G_F^2 m_W^4 m_\pi^{14}}{32\pi^5 M_X^4 M_Y^4 m_\mu^2 (m_\pi^2 - m_\mu^2)^2} \\
&\simeq 4.97 \times 10^{-18} \frac{1}{M_X^4 [GeV] M_Y^4 [GeV]}.
\end{aligned} \tag{12}$$

Putting $M_X \simeq M_Y \simeq 120$ GeV as a lower limit obtained from the LEP data analysis [17], we get $R_\pi \sim 2.3 \times 10^{-34}$. This number is much smaller than the current experimental upper limit, but it coincides with the previous theoretical evaluation without anomalous interactions included [8]. It rises a question about the mechanism for large lepton-flavour-violating pion decay mode. However, it is worth mentioning that the experimental data on R_π decrease with time, for example the 1988 data were $R_\pi \leq 8 \times 10^{-6}$, while the 1998 data are $R_\pi \leq 1.6 \times 10^{-6}$. We suggest that by adding contributions from diagrams with the exotic quarks and Higgs bosons the situation will be modified but not improved too much.

Our calculation can be analogously applied for the lepton-flavour-violating kaon decay $K^+ \rightarrow \mu^- \nu_\mu e^+ e^+$, which has an experimental branching ratio of $R_K \leq 2.0 \times 10^{-8}$. However, the main decay mode $K^+ \rightarrow \mu^+ \nu_\mu$ has only a branching ratio of 69.51 %, instead of 99.987% in the π^+ case considered here .

In summary, we have considered the lepton-flavour-violating pion decay without directly invoking lepton mixing. Our result is by twenty eight orders smaller than the current experimental upper limit. This conclusion should not be modified too much by including contributions from the exotic quarks and Higgs bosons. Hence, the mechanism for large lepton-flavour-violating pion decay mode is a mystery.

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References

- [1] Y. Fukuda *et al.*, Phys. Lett. **B433**, 9 (1998); Phys. Rev. Lett. **81**, 1562 (1998); T. Kajita, *in* Proceedings of the XVIIIth International Conference on Neutrino Physics and Astrophysics, Takayama, Japan (June 1998).
- [2] For the review see, S. M. Bilenky, C. Giunti, and W. Grimus, [hep-ph/9812360], Prog. Part. Nucl. Phys. **43**, 1 (1999).
- [3] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992); P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992); R. Foot, O.F. Hernandez, F. Pisano, and V. Pleitez, Phys. Rev. D **47**, 4158 (1993).

- [4] R. Foot, H. N. Long, and Tuan A. Tran, Phys. Rev. D **50**, R34 (1994); H. N. Long, *ibid.* D **54**, 4691 (1996); J. C. Montero, F. Pisano, and V. Pleitez, *ibid.* D **47**, 2918 (1993).
- [5] M. M. Guzzo *et al*, IPT-P.057/99, [hep-ph/9908308].
- [6] E. D. Carlson and P. Frampton, Phys. Lett. **B283**, 123 (1992); H. Fujii, S. Nakamura, and K. Sasaki, Phys. Lett. **B299**, 342 (1993); H. N. Long and T. Inami, [hep-ph/9902475], to appear in Phys. Rev. D; see also the review Y. Kuno and Y. Okada, KEK-TH 639, [hep-ph/9909265].
- [7] Particle Data Group, R. M. Barnett, *et al.*, Eur. Phys. J. **C3**, 1 (1998); Phys. Lett. **B204**, (1988).
- [8] V. A. Baranov, *et al.*, Sov. J. Nucl. Phys. **54**, 790 (1991); Yad. Fiz. **54**, 1298 (1991).
- [9] V. D. Laptev, in *Program of Experimental Investigations at the Meson Factory of the Institute of Nuclear Research of the USSR Academy of Sciences* [in Russian] (Moscow, 1984), p. 145.
- [10] D. Ng, Phys. Rev. D **49**, 4805 (1994).
- [11] P. H. Frampton and D. Ng, Phys. Rev. D **45**, 4240 (1992).
- [12] P. Das, P. Jain, and D. W. McKay, [hep-ph/9808256], Phys. Rev. D **59**, 055011 (1999).
- [13] T. Ishikawa, T. Kaneko, K. Kato, S. Kawabata, Y. Shimizu, and H. Tanaka, GRACE manual, KEK Report 92-19 (1993); H. N. Long and T. Inami, in [6].
- [14] A. Pais and S. B. Treiman, Phys. Rev. **168**, 1858 (1968); N. Cabibbo and A. Maksymowicz, *ibid.* **137**, B438 (1965); erratum *ibid* **168**, 1926 (1968); J. Bijnens, Nucl. Phys. **B337**, 635 (1990).
- [15] C. Quigg, *Gauge theories of the strong, weak and electromagnetic interactions*, Benjamin (1983); T-P. Cheng and L-F. Li, *Gauge theory of elementary particle physics*, Oxford University press, New York (1984); Q. Ho-Kim and X. Y. Pham, *Elementary particles and their interactions. Concepts and phenomena*, Springer, Berlin Heidelberg (1998).
- [16] G.P. Lepage, J. Comp. Phys. **27**, 192 (1978).
- [17] P. Frampton, Int. J. Mod. Phys. **A13**, 2345 (1998).